Radiation corrections to the ground state energy of the charged vector particle in magnetic field

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Abstract

We derive a simple representation for the polarization tensor of charged massive vector particle with gyromagnetic ratio $g = 2$ in a magnetic field $B = const$, averaged over the ground state. This expression corresponds to the Demeur formula in QED. We investigate the ground state energy at the threshold of the tree level instability, $B \to B_c = m^2/e$. In the standard model, it is found that the energy determined from the Schwinger-Dyson equation is real in this limit for the values of the Higgs boson mass $m_H \geq m_Z$, that ensures stability of the W-boson spectrum. In the effective ρ - meson electrodynamics, the electromagnetic radiation corrections shift the threshold of the instability to weaker fields $B < B_c^\rho$. Some peculiarities of the effective theory are discussed.

1 Introduction

Recently, physics of charged vector particles in strong magnetic fields has obtained new stimulus for investigation. First of all, this concerns ρ-meson physics. For these particles, the effective Lagrangians describing electromagnetic interactions have been derived in different approaches [1], [2], [3], [4]. As an important characteristic, the gyromagnetic ratio for charged $\rho^\pm$- mesons, $g = 2$, was determined, as it holds for the W- bosons and charged gluons. Moreover, the effective Lagrangian for ρ-mesons is very similar to the standard model one. Hence, interesting phenomena investigated already in the electroweak theory [5], [6] may have relevance to the ρ-mesons, also. In Refs. [7], [8], [9] the properties of the electroweak vacuum versus the QCD vacuum in strong magnetic fields of the order $eB \geq m_\rho^2$, where m_ρ - particle mass, were investigated and the superconducting state having a structure similar to Abrikosov's lattice observed.

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The key point in describing of this new vacuum is the spectrum of charged vector particle with gyromagnetic ratio $g = 2$,

$$p_0^2 = p_{||}^2 + m^2 + (2n+1)eB - 2e\sigma B, \qquad (n = 0, 1, ..., \ \sigma = 0, \pm 1), \qquad (1)$$

in a homogeneous magnetic background, B, described by the potential

$$A_\mu^{ext.} = B x_1 \ \delta_{\mu 2}, \qquad (2)$$

where $p_{||}$ is a momentum component along the field direction, e - electric charge. Here, a tachyon mode is present in the ground state ($|t> \quad = |n = 0, \sigma = +1>$) for the field strength $B \geq B_c = m^2/e$. Condensation of this mode, which happens due to nonlinearity of fields, results in the inhomogeneous vacuum state mentioned. It worth to note that the mass term in eq. (1) has different origin in the electroweak and ρ-meson models. In the former case, the mass is expressed through a scalar condensate, and in the latter model it is generated due to strong forces acting between quarks.

The main idea of the present investigation grounds on the fact that in the external field presence the spectrum of particle should be determined by expecting the poles of the Schwinger-Dyson equation which accounts for the radiation corrections. This is because close to the threshold of instability B_c the effective mass of the ground state is completely determined by the polarization tensor $\Pi(B, p_{||})$: $m_{eff}^2 =< t|m^2 - eB - \Pi(B \to B_c, p_{||} = 0)|t >$. Hence, positive value of $< t|\Pi(B \to B_c, p_{||} = 0)|t >$ stimulates the condensation, and negative - prevents it. In any case, the radiation corrections influence considerably the parameters of the particle in strong fields and have to be taken into account.

In fact, this problem has been investigated already for the W- bosons in the Georgy-Glashow model of electroweak interactions [11], [12]. As it was found, the result depends on the value of the Higgs boson mass m_H. For heavy Higgs particle, $K = m_H/m_W \geq 1.2$, the contribution of the gauge field sector dominates and the spectrum stabilization takes place. For light Higgs particle, $K < 1.2$, the instability was found. Since the values of K, which increases instability determined from the tree-level spectrum, admit sufficiently heavy $m_H \sim m_Z$, it was reasonable to conclude that W-boson condensation has to happen, as it is described on the base of classical field equations [5], [6]. However, this problem was not investigated in detail for the standard model. Here, the presence of Z-bosons may change the situation.

Other remark, from our present day knowledge, the noted results on the influence of radiation corrections are incomplete. This is mainly because of the absence at that time the adequate representation for the ground state projection of the W-boson polarization tensor, similar to the Demeur formula for electron in magnetic field [10]. As a result, a bulky general type expression was investigated and estimated in Refs. [11], [12].

In the ρ-meson case the situation is more complicated. This is because the effective Lagrangian [1], [7],

$$L_\rho = -\frac{1}{2}(D_[\mu, \rho_\nu])^+ D^[\mu, \rho_\nu] + m_\rho^2 \rho_\mu^+ \rho^\mu - \frac{1}{4}\rho^{(0)}_{\mu\nu}\rho^{(0)\mu\nu} + \frac{1}{2}m_\rho^2 \rho^{(0)}_\mu \rho^{(0)\mu} + \frac{e}{2g_s}F_{\mu\nu}\rho^{(0)}_{\mu\nu}, \quad (3)$$

where $D_\mu = \partial_\mu + ig_s\rho^{(0)}_\mu - ieA_\mu$ is the covariant derivative, $g_s \sim 6$ is the $\rho\pi\pi$ coupling, A_μ is the photon field with the field strength $F_{\mu,\nu} = \partial_{[\mu,A_\nu]}$, the field $\rho = 1/\sqrt{2}(\rho^{(1)}_\mu - i\rho^{(2)}_\mu)$ and $\rho^{(0)}_\mu = \rho^{(3)}_\mu$ correspond, respectively, to the charged and neutral vector mesons with the masses, m_ρ, and $\rho^{(0)}_{\mu\nu} = \partial_[\mu, \rho_\nu] - ig_s[\rho^+_\mu, \rho_\nu]$, actually coincides with the one for the standard model in the unitary gauge, if one takes the Z-boson mass, m_Z, as the mass of neutral ρ-meson and changes the corresponding gauge coupling to the strong coupling, g_s. Formally, a theory with such type Lagrangian is not renormalizable one (even for small g_s). So, electromagnetic radiation corrections must be treated in a special way, here. Remind, in the standard model, the divergences of non-renormalizable type coming from diagrams generated by the three-leg vertexes (having a coupling e) are canceled by the contributions of the contact vertexes (having the coupling $\sim e^2$) that is ensured by the nonAbelian gauge symmetry and partially by the contributions of the diagrams with the Higgs field. These terms absent in eq. (3). Nevertheless, it is a possibility to estimate the role of electromagnetic radiation corrections near the threshold of instability, $B \to B_c$, for this case, also. The idea is to take into consideration the relevant expression obtained in the standard model. Then, our expectation is that in this limit a number of specific diagrams dominate in both cases. Since the form of electromagnetic interactions is the same, and the effective Lagrangian was derived from the renormalizable theory, one can believe that the role of radiation corrections can be clarified.

In Ref. [13], we calculated the ground state projection of the $SU(2)$ polarization tensor for charged gluons in Abelian chromomagnetic field like eq. (2) and investigated it for high temperature. Therein the simple expression for this function was derived which just corresponds to the Demeur formula for electron in magnetic field. The obtained results (eqs. (17), (22) in Ref. [13]) can be modified to find the ground state energy for charged massive vector particle. Of course, a number of other contributions should also be added in different models. In what follows, we apply the results of Ref. [13] to calculate the ground state energy, $< t|\Pi(p_{||}, B)|t >$, for the W-boson, accounting for the one-loop diagrams. This expression is investigated in the limit of $B \to B_c$. We show that the energy calculated from the Schwinger-Dyson equation remains real for realistic masses of Higgs particle. Thus, radiation corrections prevent the instability of the vacuum. We show that technically this is due to the dominant contributions of the W- boson tadpole diagrams. Hence, one can believe that the same effect is predominated in the ρ-meson electrodynamics. However, this may not be the case

because of the strong coupling g_s in the effective theory. Here, the pure electromagnetic corrections act to decrease the field strength, $B = B' < B_c$, when the instability of the vacuum has to start.

In the next section we calculate the W- boson polarization tensor and get it mean value in the ground state of the spectrum (1). In sect. 3. we investigate the limit of $B \to B_c$. In sect. 4 the case of charged ρ- mesons in strong magnetic fields is considered. General conclusions and discussion are given in the last section.

2 W- boson polarization tensor

In what follows, we use Euclidean space-time and the representation of the polarization tensor for gluons given in [14]. It is reasonable to rewrite color gluon field, V_μ^a $(a = 1, 2, 3)$, in terms of charged, $W_\mu^\pm = \frac{1}{\sqrt{2}}(V_\mu^1 \pm iV_\mu^2)$, and neutral, $V_\mu^3 = A_\mu^3$, components. In momentum representation, the initial expression reads

$$\begin{aligned}\Pi_{\lambda\lambda'}(p) \quad &= \quad \int \frac{dk}{(2\pi)^4}\ \{\Gamma_{\lambda\nu\rho}G_{\nu\nu'}(p-k)\Gamma_{\lambda'\nu'\rho'}G_{\rho\rho'}(k) \\ &\quad + (p-k)_\lambda G(p-k)k_{\lambda'}G(k) + k_\lambda G(p-k)(p-k)_{\lambda'}G(k)\} \\ &\quad + \Pi_{\lambda\lambda'}^{\text{tadpol}}\ , \end{aligned} \tag{4}$$

where the second line results from the ghost contribution and the tadpole contribution is given by

$$\Pi_{\lambda\lambda'}^{tadpol} = \int \frac{dp}{(2\pi)^4}\ \{2G_{\lambda\lambda'}(p) - \delta_{\lambda\lambda'}G_{\rho\rho}(p) - G_{\lambda\lambda'}(p)\}\,. \tag{5}$$

The contributions of the charged tadpole diagrams are taken into consideration. Only these tadpoles are relevant to the problem of interest. The vertex factor,

$$\Gamma_{\lambda\nu\rho} = (k-2p)_\rho\ \delta_{\lambda\nu} + \delta_{\rho\nu}(p-2k)_\lambda + \delta_{\rho\lambda}(p+k)_\nu\ , \tag{6}$$

completes the description of the vector part of the polarization tensor. These formulas hold also in a background field, provided the corresponding expressions for the propagators are used. We take a homogeneous magnetic background field in the representation given in eq. (2). In this case the operator components fulfill the commutation relation

$$[p_\mu, p_\nu] = i\,eF_{\mu\nu} \quad (F_{12} = B). \tag{7}$$

In what follows, where it will be not misleading, we write B instead eB or even put $B = 1$, for short. In above formulas (4) and (5) we omitted the coupling factors e^2. These factors as well as other factors proper to different models of interest can be accounted for in the final expressions. Below, we will also use the

notations $l^2=l_3^2+l_4^2$ and $h^2=p_1^2+p_2^2$, where we write l_3 and l_4 for the momenta in parallel to the background field $p_{||}=l_3$ and imaginary time, respectively. Other information relevant to the massless case is given in Refs. [13], [14].

To obtain the results for the electroweak sector, one has to take into account the masses of the W- , Z- and Higgs bosons, and add the contributions of the latter two particles.

First we incorporate the masses in the representation of the polarization tensor as given in eq. (51) of Ref. [14]. It results from the proper time representation of the propagators,

$$\begin{aligned} G(p-k) &= \int\limits_0^\infty ds\; e^{-sm^2}e^{-s(p-k)^2}, \qquad G(k)=\int\limits_0^\infty dt\; e^{-tM^2}e^{-tk^2}, \\ G_{\lambda\lambda'}(p-k) &= \int\limits_0^\infty ds\; e^{-sm^2}e^{-s(p-k)^2}E_{\lambda\lambda'},\;\; E_{\lambda\lambda'}=e_{\lambda\lambda'}^{-2ieFs}, \end{aligned} \tag{8}$$

for charged and neutral particles, and integration over k in eq. (4). The mass M is, $M=0,m_Z,m_H$ for photon, Z- and Higgs boson, correspondingly.

The representation for the $SU(2)$ sector of the standard model (W-bosons, massive ghosts and photons) is obtained in terms of the integral over the parameters s and t,

$$\Pi_{\lambda\lambda'}=\int_0^\infty ds\int_0^\infty dt e^{-sm^2}\;\Theta(s,t)\left(\sum_{i,j}M_{\lambda\lambda'}^{i,j}+M_{\lambda\lambda'}^{\rm gh}\right)+\;\Pi_{\lambda\lambda'}^{\rm tadpol} \tag{9}$$

with

$$\Theta(s,t)=\frac{\exp(-H)}{(4\pi)^2(s+t)\sqrt{\Delta}}. \tag{10}$$

Here the following notations are used:

$$\begin{aligned} H &= \frac{st}{s+t}\,l^2+m(s,t)(2n+1)B\,, \\ m(s,t) &= s+\frac{1}{2}\ln\frac{\mu_-}{\mu_+}\,, \\ \Delta &= \mu_-\,\mu_+, \\ \mu_\pm &= t+\sinh(s)e^{\pm s}, \end{aligned} \tag{11}$$

which are equivalent to eqs. (23-26) in [14]. The sum over i,j in (9) follows the subdivision introduced in [14] and the functions $M_{\lambda\lambda'}^{i,j}$ are given by eq. (53) in [14].

Now we take the tachyonic projection of $\Pi_{\lambda\lambda'}$, eq. (9). In doing so we note especially $n=0$ (for $B=1$) and the function Θ simplifies,

$$\Theta(s,t)_{|h^2=1}=\frac{\exp(-\frac{st}{s+t}l^2-s)}{(4\pi)^2(s+t)\mu_-}. \tag{12}$$

For the projection of the functions $M^{i,j}_{\lambda\lambda'}$ we use representation (55) in [14]. Calculation of these terms is given in the appendix of Ref. [13]. At this place we mention that under the tachyonic projection we get directly a representation suitable for further calculations. The presence of particle masses is reflected in a simple factor in the integrand of eq. (9) and not influenced any computation procedures applied in the massless case.

Note that the expression (9) is calculated in the Feynman-Lorentz-t'Hooft gauge

$$P_\mu W^-_\mu - m\phi^- = iC^-, \tag{13}$$

in which the mass of charged ghost, $C^\pm$, and Goldstone, $\phi^\pm$, fields equals to the W-boson mass m.

Detailed calculations of $< t|\Pi|t >$ are given in Ref. [13] and not modified for $m \neq 0$. Only the contribution from $M^{33} + M^{gh}$ requires an additional consideration. As it is shown in Ref. [14], eqs. (87) - (89), this part can be written in the form,

$$M^{33+gh}_{\lambda\lambda'} = -\int_0^\infty ds\ dt\ e^{-s\,m^2}\Big(\delta_{\lambda\lambda'}\frac{\partial\Theta(s,t)}{\partial s} + E_{\lambda\lambda'}\frac{\partial\Theta(s,t)}{\partial t}\Big), \tag{14}$$

where $\Theta(s,t)$ is the function in eq. (10) and the matrix $E_{\lambda\lambda'} = e^{-2isF}_{\lambda\lambda'}$. These combining into derivatives allow for carrying out one of the parameter integrations. Using $\langle t \mid \delta_{\lambda\lambda'} \mid t\rangle = 1$, $\langle t \mid E_{\lambda\lambda'} \mid t\rangle = e^{2s}$,

$$\Theta(s=0,t) = \frac{1}{t^2}, \qquad \Theta(s,t=0) = \frac{1}{s\sinh(s)}, \tag{15}$$

and integrating by part we get in the projection

$$\begin{aligned}\int ds\,dt\ e^{-s\,m^2}\langle t \mid M^{33+gh}\,\Theta(s,t) \mid t\rangle &= \frac{1}{(4\pi)^2}\int\frac{dq}{q}\left(\frac{1}{q} + \frac{e^{-q\,m^2}e^{2\,q}}{\sinh(q)}\right)\\ &-\frac{m^2}{(4\pi)^2}\int_0^\infty ds\ dt\ e^{-sm^2}\frac{e^{-s}e^{-\frac{st}{s+t}l^2}}{(s+t)\mu_-},\end{aligned} \tag{16}$$

where in the last line the function (12) is substituted. To complete this part, we write down the remaining (except M^{33+gh}) terms coming from the main diagram, (4),

$$\langle t \mid \sum_{ij}\bar{M}^{ij} \mid t\rangle = \frac{4}{\mu_-} + 4\frac{s+t\,e^{2s}}{s+t}\ l^2, \tag{17}$$

and hence

$$\langle t \mid \Pi(\sum_{ij}\bar{M}^{ij}) \mid t\rangle = \frac{1}{(4\pi)^2}\int\frac{ds\,dt}{s+t}\,e^{-sm^2}\ \left[\frac{4}{\mu_-} + 4\frac{s+t\,e^{2s}}{s+t}\ l^2\right]\ \frac{e^{-\frac{st}{s+t}l^2-s}}{\mu_-}. \tag{18}$$

Here, $\bar{M}^{ij}$ reminds about the omitted terms. The contributions from the tadpoles, (5), take the form

$$\langle t \mid \Pi^{\mathrm{tp}} \mid t\rangle = -\frac{1}{(4\pi)^2}\int \frac{dq}{q}\, e^{-q\, m^2}\left(\frac{2+\cosh(2q)+3\sinh(2q)}{\sinh(q)}\right). \qquad (19)$$

Then we have to add the contributions coming from charged Goldstone bosons. As computations shown, one term coincides up to the sign with the last line in eq. (16), and exactly cancels in the total, as it should be in the renormalizable gauge (13). Other contribution is the tadpole one coming from the contact vertex $\sim e^2\, W^+_\mu W^-_\mu \phi^+\phi^-$. This term up to the factor $-1/4$ coincides with the first term in eq. (19). Thus, the expressions (16) (except the second line), (18) and (19), corrected due to the noted tadpole contribution, represents the electromagnetic part of the W- boson polarization tensor in the projection to the lower, the tachyonic state. It corresponds to the Demeur formula for electron in magnetic field in QED (see Ref. [10], eq. (59)).

Now, we turn to the contributions of the Z- boson sector, calculated in the gauge

$$\partial_\mu Z_\mu - i\, m_Z \phi^Z = C^Z, \qquad (20)$$

where ϕ^Z and C^Z present the Goldstone and ghost fields having the mass m_Z. This part can be expressed by using the obtained expressions eqs. (16), (18). Actually, according to (8), one has to introduce in these formulas the mass factor $e^{-t\, m_Z^2}$. The contribution of the Golstone field ϕ^z and the term in the second line of (16) are canceled again. But now, after integration by part over the parameter t in eq. (14), new term appears. The sum of contributions from Goldstones and M_Z^{33+gh} looks as follows,

$$\int ds\, dt\; e^{-s\, m^2 - t\, m_Z^2} \langle t \mid M^{33+gh}_{Z.Gb}\,\Theta(s,t)\mid t\rangle = \frac{1}{(4\pi)^2}\int \frac{dq}{q}\left(\frac{e^{-q\, m_Z^2}}{q} + \frac{e^{-q\, m^2} e^{2\, q}}{\sinh(q)}\right)$$
$$-\frac{m_Z^2}{(4\pi)^2}\int_0^\infty \frac{ds dt\; e^{-\frac{st}{s+t} l^2} e^s}{(s+t)\mu_-} e^{-s\, m^2 - t\, m_Z^2}, \qquad (21)$$

where the factor $e^{2\, s}$ in the second line appears from $E_{\lambda\lambda'}$ in eq. (14). Thus, the contribution from the Z- sector is given by eq. (21) and eq. (18) with additional factor $e^{-t\, m_Z^2}$ in the integrand.

Then, we restore the couplings and the dimensionality in the obtained expressions. Remind that in actual calculations we put $eB = 1$ and therefore the proper-time parameters s, t, q became dimensionless. In fact, this means that we measure them, as well as the masses, in units of eB. Thus, to recover the dimensionality one has to substitute $M^2 \to M^2/(e\, B)$, $l^2 \to l^2/(e\, B)$ and extra total factor $(e\, B)$ coming from the $\Theta(s,t)$ in eqs. (10), (12). For the electromagnetic sector, we have to introduce the factor e^2 in eqs. (16), (18) and the factor

$g^2 = e^2/\sin^2\theta$ for the tadpole contributions eq. (19). For the Z-boson sector, the overall factor in eqs. (16), (18) is $e^2\cot^2\theta$. Here θ is the Weinberg angle.

The contribution of the Higgs boson sector is given by two diagrams and reads,

$$\begin{aligned}\Pi^H_{\lambda\lambda'}(p) &= \int \frac{dk}{(2\pi)^4}\Big\{(2k-p)_\lambda G(p-k,m^2)(2k-p)_{\lambda'}G(k,m_H^2) \\ &\quad + 4\,m^2\,G_{\lambda\lambda'}(p-k,m^2)\;G(k,m_H^2)\Big\}, \qquad (22)\end{aligned}$$

where we marked the mass of the particle. Again, we have to use the representation (8) and then integrate over k. In the ground state projection, the first line simplifies considerably because the condition $p_{\lambda'} \mid t\rangle_{\lambda'} = 0$ holds. So, only the term $4\,k_\lambda\,k_{\lambda'}$ contributes. The corresponding term up to a factor coincides with the term $\langle t \mid M^{11} \mid t\rangle$, eq. (51) in Ref. [13]. The second line equals just to $\langle t \mid E_{\lambda\lambda'} \mid t\rangle\Theta(s,t)_{h^2=1}$ (see eq. (12)).

Thus, for the scalar sector we obtain,

$$\begin{aligned}\langle t \mid \Pi^H(p) \mid t\rangle &= \frac{g^2}{(4\pi)^2}\Bigg\{e\,B\int\limits_0^\infty \frac{dsdt\; e^{-\frac{st}{s+t}\frac{l^2}{eB}}}{(s+t)\mu_-^2} e^{-(s(\,m^2/(eB)+1)+t\,m_H^2/(eB))} \\ &\quad + m^2\int\limits_0^\infty \frac{dsdt\; e^{-\frac{st}{s+t}\frac{l^2}{eB}}}{(s+t)\mu_-} e^{-(s(\,m^2/(eB)-1)+t\,m_H^2/(eB))}\Bigg\}, \qquad (23)\end{aligned}$$

where all the necessary factors are substituted. The expressions (16), (18), (19), (21) with corresponding factors and eq. (23) give the non-renormalized mean value of the W- boson polarization tensor in the ground state $\mid t\rangle$ in the standard model. Its renormalization is fulfilled in a usual way by subtracting of the terms

$$\begin{aligned} c.t._1 &= \langle t \mid \Pi(p^2,B,m^2)_{|B=0} \mid t\rangle, \\ c.t._2 &= \langle t \mid \frac{\partial \Pi(p^2,B,m^2)}{\partial p^2}(l^2 = eB - m^2)_{|B=0} \mid t\rangle \qquad (24)\end{aligned}$$

on the mass shell of the spectrum (1) in the ground state $l^2 = eB - m^2$ (see Refs. [11], [13] for details). These counter terms are divergent at the lower limit $s,t = 0$.

On the base of these formulas, different kind studies can be carried out. In the next section, we investigate the ground state energy in the limit of $B \to B_c$. For this case, the main contributions come from the upper limit of integrations over the proper time parameters. So, the renormalization is not important.

3 Ground state energy at $B \sim B_c$

Let us consider the limit of the field strength $B \to m^2/e$ for calculated expressions. In this case, a number of terms is divergent at the upper limit of integration

because of the smallness of the "effective tree-level mass", $\Delta = m^2 - eB$, which enters the cutting factor $e^{-s\Delta}$ going to unit for $\Delta \to 0$, and integrals diverge. These are the last term in the first line of eq. (16) and similar term in eq. (21), two last terms in eq. (19) and the terms in the second lines in eqs. (21) and (23). The sum of calculated diagrams obtains the overall factor, g^2, of $SU(2)_w$ gauge group, and the mass m_Z has to be substituted by m, due to the relation $e^2 = g^2 \sin^2\theta$. They give dominant contributions and should be accounted for.

As a result, when all the relevant terms are gathered together, two types of integrals contribute in this limit,

$$\epsilon_t^2 = \langle t \mid \Pi \mid t \rangle = \frac{g^2}{(4\pi)^2}(I^{(1)} + I^{(2)}). \tag{25}$$

First is one parametric,

$$I^{(1)} = -2eB \int\limits_c^\infty \frac{dq}{q}\; e^{-q\,(m^2/(eB)-1)}, \tag{26}$$

where c is a constant of order 1. The second integral is two parametric,

$$I^{(2)} = m^2 \int\limits_0^\infty dsdt \frac{e^{-(m^2/(eB)-1)\frac{s^2}{s+t}}}{(s+t)\mu_-}\Big(e^{-t\,m_H^2/(eB)} - e^{-t\,m_Z^2/(eB)}\Big). \tag{27}$$

In the last expression, we used the relation $l^2 = eB - m^2$. Both of these integrals can be easily estimated. The first is negative and equals,

$$I^{(1)}(B)\mid_{B\to B_0} = -2eB\log\Big(\frac{1}{m^2/(eB)-1}\Big) + O(1). \tag{28}$$

The sign of I_2 depends on the relation between the masses m_H and m_Z. If $m_H \geq m_Z$, the second term in eq. (27) is dominant and integral is negative. Otherwise, it is positive. In the special case, $m_H = m_Z$, $I_2 = 0$. We get for the leading term,

$$I^{(2)}(B)\mid_{B\to B_0} = m^2\log\Big(\frac{1}{m^2/(eB)-1}\Big)[\log\Big(\frac{2m^2+m_H^2}{2m^2+m_z^2}\Big) + \log\frac{m_Z^2}{m_H^2}] + O(1). \tag{29}$$

Thus, in the standard model the radiation correction to the ground state energy at the threshold of instability, $B = B_c$, is negative for realistic values of the masses $m_H > m_Z$. Note that in the Georgy-Glashow model Z- boson absences and $I_2 > 0$.

On the base of these calculations, we conclude that radiation corrections act to stabilize the tree-level spectrum (1). Really, if one considers the pole of the Schwinger-Dyson operator equation taken in the ground state, $< t|D|t >^{-1} = \langle t \mid m^2 + l_3^2 - B - \Pi(B, m^2, \Delta \to 0) \mid t\rangle$, then the positivity of the "effective mass squared", $m_{eff.}^2(B) = m^2 - eB + \epsilon_t^2$, follows. To complete this part, we note that the fermion contribution is independent of the unstable mode and does not contribute in this limit to the effective mass $m_{eff.}^2(B)$ in one-loop approximation.

4 Radiation spectrum of $\rho^{\pm}$-meson at $B \sim m_\rho^2/e$

In this section, within the effective theory described by the Lagrangian (3), we investigate the influence of electromagnetic radiation corrections on the ground state of the charged $\rho^{\pm}$- mesons at the threshold of the instability $B \sim B_c^\rho = m_\rho^2/e$. In fact, we are going to discuss to which extend the results obtained above for the W- boson have relevance in the case of the effective theory.

The expression (3) almost coincides with the standard model Lagrangian in the unitary gauge. Minor difference consists in the much less relative splitting of ρ^0- and $\rho^{\pm}$- masses as compared to the $W^{\pm}$- and Z- boson ones, and the absence of the scalar field contribution in (3). Formally, in the unitary gauge the standard model looks as a non-renormalizable one. Renormalizability is ensured due to gauge invariance and the presence of the Higgs scalar field. More essential difference, however, consists in the large value of the effective strong coupling constant, $g_s \sim 6$, for ρ - mesons. This fact seems for us the most important. It could prevent a straightforward extension of the results obtained in the fundamental theory to the effective model. Remind [1] that in the effective theory a small difference between the masses of the neutral and charged mesons, $m_{\rho_0} - m_{\rho_\pm} = 1.MeV$, is related with large difference between the values of e, and g_s.

To make a comparison, let us consider for a moment a toy model with sufficiently small $g_s \leq 1$. In this case, one can consider eq. (3) as the standard model with very small Weinberg angle $\theta_{tm} \sim 0$, and, hence, $m_Z^{tm} = m/\cos\theta_{tm} \sim m$. Further, one can transform the Lagrangian (3) into the gauge (13) and obtain the results given in eqs. (16), (21), (19). The contributions coming from the Higgs boson loops absence, now. Then, taking into account the results from previous section, we find that radiation corrections stabilize the spectrum of charged particle at the threshold of instability B_c^{tm}. Thus, if it is possible to apply the results obtained for small g_s to the case of $g_s \geq 1$, the conclusion about the spectrum stabilization is in order. These arguments look reasonable, because the negative sign of the radiation energy is completely determined by the sign of the tadpole diagrams of charged vector particles. Hence, one can believe that the sign does not change when the value of g_s becomes enough large.

On the other hand, for the effective theory derived due to strong interactions of quarks as eq. (3) and containing strong coupling, the account for the radiation corrections requires a special consideration. In any case, it is consistent to calculate the radiation electromagnetic corrections, but the effective strong interactions of ρ - mesons have to be taken into account at tree level, only. The effective Lagrangians are used to describe phenomena at low energies. As a rule, they are obtained by means of some kind integration over fast varying variables. In the case under consideration, the ρ- mesons were obtained as effective fields appeared after integration over quark fields at high energies [1], [2], [3], [4]. Hence,

formally, to investigate the role of radiation corrections, one has to return back to the initial Lagrangian, calculate radiation corrections and then derive new effective Lagrangian for low energies. This point, of course, requires further analysis in the frame of the effective Lagrangian approach, that is out of the scope of the present paper.

In the case of electromagnetic radiation corrections, the situation is changed to the opposite general conclusion as compared to the one in previous section. Really, if we take into account the terms coming due to electromagnetic field in the loop given in the first line of eq. (16), the value

$$\epsilon^2_{em} = \frac{e^2}{(4\pi)^2} \int\limits_0^\infty d\,q\, e^{-q(m^2_\rho/(eB))} \Big(\frac{1}{q \sinh(q)} + 2 \frac{e^q}{q} \Big) \tag{30}$$

follows. In the limit of $B \to B^\rho_c$ the second term gives dominant contribution,

$$\epsilon^2_{em}(B) \mid_{B \to B^\rho_c} = \frac{e^2}{8\pi^2}\, eB \log\Big(\frac{1}{m^2_\rho/(eB) - 1} \Big) + O(1). \tag{31}$$

We see now, $\epsilon^2_{em}(B)$ is divergent and positive at $B \sim B^\rho_c$. So, the electromagnetic radiation corrections shifts the threshold of instability to weaker fields, $B = B' < B^\rho_c$, and stimulate the phase transition - condensation of charged ρ - mesons. They modify the parameters of the condensate because the effective ρ - meson mass is $m^2_{eff.}(B) = m^2_\rho - eB - \epsilon^2_{em}(B)$. It is also important that the ultraviolet behavior of the effective theory, determined at the lower limit in the proper-time representation used, is not essential in this case. The leading term in ϵ^2_{em} is completely determined by the behavior of eq. (30) at the upper limit, $q \to \infty$.

5 Discussion and conclusions

We derived simple convenient representation for the mean value of the W- boson polarization tensor in external magnetic field calculated in the ground state of the tree-level spectrum (1). It corresponds to the Demeur formula for electron in magnetic field in QED. As application, we investigated its behavior at the threshold of instability $B \to B_c$. We have found, this value is negative and therefore the effective mass determined within the Schwinger-Dyson equation $m^2_{eff.}(B) = \langle t \mid m^2 - eB - \Pi(B \to B_c) \mid t \rangle$ remains positive, that prevents the vacuum instability in strong fields. As it is occurred, the dominant contribution in the projection comes from the charged tadpole diagrams. This result is similar to the one obtained already in the Georgy-Glashow model [11], [12]. In the latter case, however, there exist the range of not heavy Higgs boson mass for which the radiation corrections shift the threshold of instability to the weaker than B_c field strengths, and increase instability. In the standard model, such type domain absences for realistic values of $m_H \geq m_Z$.

Now, we also take into consideration that the particle masses are determined through the effective potential (EP) of scalar field $V(\phi_c, B)$ and depend on B: $m(B) = \frac{1}{2} g \phi_c(B), m_Z(B) = m(B)/\cos\theta$ and for Higgs particle $m_H^2(B) = \partial^2 V(\phi_c, B)/\partial\phi_c^2$ (taken in the minimum $\phi_c = \delta(B)$ of the potential). Since the minimum position $\delta(B)$ and the curvature in the minimum are different in general and strongly dependent on the field strength, the role of the scalar field and the gauge boson radiation corrections may change. For instance, the equality $m_H^2(B) = m_Z^2(B)$ could never be realized. Therefore, the cancelation of I_2 term in eq. (27) never takes place, even if in the classical potential this equality is implemented by construction. As a conclusion, we note that in strong external fields the tree level result could not be adequate to real situation, and field dependence of parameters may in an essential way influence actual vacuum state.

In applications to the effective ρ - meson electrodynamics, we have discussed two possibilities. First is based on the properties of the Lagrangian (3) and its similarity to the standard model Lagrangian in the unitary gauge. Hence, it is possible to conclude that, as in the latter case, the vacuum stabilization also takes place. The main argument here is based on the observation that the dominant contribution to the ground state energy comes from the tadpoles, and this fact hardly changes for large values of the ρ - meson effective coupling g_s. Second possibility is based on the requirement that in the effective ρ - meson model strong interactions should be accounted for in tree approximation, only. The electromagnetic radiation corrections to the energy now stimulate the instability in strong magnetic fields and modify the parameters of the ρ - meson condensate as compared to the three-level results obtained in Refs. [7]- [9]. This picture seems more realistic. Although, further analysis in the frame of the effective Lagrangian approach is in order.

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